Answer Key

PC11 Chapter 2 Absolute Values and Radicals Review

1. Evaluate each expression

a) $\sqrt{(5.1-2.3)^2}-|5.1-2.3|$

$= \sqrt{7.84} - 2.8$

$= 2.8 - 2.8$

$\boxed{= 0}$

b) $|5-4|(5+4)-2(5+4)$

$= |1|(9) - 10 - 8$

$= 1(9) - 18$

$= 9 - 18 \quad \boxed{= -9}$

c) $\dfrac{|2-(-8)|}{|3|-|-2|}$

$= \dfrac{|10|}{3-2}$

$= \dfrac{10}{1} \quad \boxed{= 10}$

d) $\dfrac{2}{3}\left|-\dfrac{5}{8}-\dfrac{1}{4}\right|$

$= \dfrac{2}{3}\left|-\dfrac{7}{8}\right|$

$= \dfrac{2}{3}\left(\dfrac{7}{8}\right)$

$= \dfrac{14}{24} \quad \boxed{= \dfrac{7}{12}}$

2. Evaluate

a) $|x^2+6x-5|, x=2$

$= |(2)^2+6(2)-5|$

$= |4+12-5|$

$= |11| \quad \boxed{= 11}$

b) $|x^3+5x-2|, x=-3$

$|(-3)^3+5(-3)-2|$

$= |-27-15-2|$

$= |-44| \quad \boxed{= 44}$

3. Arrange from greatest to least

a) $-|3|, |-3|, -|-2|, |-(-2)|$

$(-3), (3), (-2), 2.$ → Least to greatest: $\{-|3|, -|-2|, |-(-2)|, |-3|\}$

Greatest to least: $\{|-3|, |-(-2)|, -|-2|, -|3|\}$

b) $-|2.\bar{5}|, |-2.\bar{5}|, -|2.5|, |-2.5|$

$(-2.\bar{5}), (2.\bar{5}), (-2.5), (2.5)$: Greatest to least: $\{|-2.\bar{5}|, |-2.5|, -|2.5|, -|2.\bar{5}|\}$

4. Two integers, a and b, are between -5 and 5. The sum of the absolute values of the integers is 4. Give all possible values.

$|a| + |b| = 4$

Possible $a = 1, 2, 3, 4, 0, -1, -2, -3, -4$

Possible $b = 1, 2, 3, 4, 0, -1, -2, -3, -4$.

6. Write as an entire radical

a) $2\sqrt{3}$ $\quad \sqrt{12}$

b) $-4\sqrt{5}$ $\quad -\sqrt{80}$

c) $3\sqrt{4}$ $\quad \sqrt{36}$

d) $\frac{2}{3}\sqrt{5}$ $\quad \sqrt{\frac{20}{9}}$

e) $2\sqrt[3]{3}$ $\quad \sqrt[3]{24}$

f) $-4\sqrt[3]{5}$ $\quad \sqrt[3]{-320}$

7. Write each radical in simplest form. For what values of the variables is the radical defined?

a) $(\sqrt{3a^2b})(\sqrt{6ab^5})$ $a \geq 0$, $a \in \mathbb{R}$; $b \geq 0$, $b \in \mathbb{R}$

$= \sqrt{18a^3b^6}$

$= \sqrt{9 \cdot 2 \cdot a^2 \cdot a\,(b^3)^2}$

$= 3ab^3\sqrt{2a}$

b) $(4x\sqrt{10xy})(3y\sqrt{2x})$ $x \geq 0$, $x \in \mathbb{R}$; $y \geq 0$, $y \in \mathbb{R}$

$= 12xy\sqrt{20x^2y}$

$= 12xy\sqrt{4 \cdot 5x^2y}$

$= 24x^2y\sqrt{5y}$

c) $(2x\sqrt[3]{2y^4})(x^2\sqrt[3]{4y^2})$ $y \in \mathbb{R}$

$= 2x^3\sqrt[3]{8y^6}$

$= 2(2)x^3y^3$

$= 4x^3y^3$

d) $(ab\sqrt[3]{2ab^2})(3a\sqrt[3]{4a^2b^2})$ $a \geq 0$; $a \in \mathbb{R}$; $b \in \mathbb{R}$

$= 3a^2b\sqrt[3]{8a^3b^4}$

$= 3a^2b(2)(a)(b^2)\sqrt{a}$

$= 6a^3b^3\sqrt{a}$

e) $\dfrac{9x^2\sqrt{x^2y^5}}{3x^5\sqrt{x^6y}}$ $x \in \mathbb{R}$, $y \geq 0$, $\in \mathbb{R}$

$= \dfrac{3y^2}{x^5}$

f) $\dfrac{\sqrt[3]{81x^2y^5}}{\sqrt[3]{x^5y}}$ $x \in \mathbb{R}$, $y \in \mathbb{R}$

$= \dfrac{3y}{x}\sqrt[3]{3y}$

8. Simplify each radical. For what values of the variables is the radical defined?

25) $-4\sqrt{216x^2y^2z}$ $\begin{cases} x^2 \geq 0 \\ y^2 \geq 0 \\ z \geq 0 \end{cases}$ $x \in \mathbb{R}$, $y \in \mathbb{R}$, $z \in \mathbb{R}$

$= -4\sqrt{36 \cdot 6 \cdot x^2y^2z}$

$= -4(6)|x| \cdot |y|\sqrt{6z}$

$= \boxed{-24xy\sqrt{6z}}$

26) $-3\sqrt{24a^4b^2c^3}$ $\begin{cases} a \in \mathbb{R} \\ b \in \mathbb{R} \\ c \geq 0, \in \mathbb{R} \end{cases}$

$= -3\sqrt{4 \cdot 6 \cdot (a^2)^2\, b^2 \cdot c^2 \cdot c}$

$= -3(2) \cdot a^2 \cdot b \cdot c\sqrt{6c}$

$= \boxed{-6a^2bc\sqrt{6c}}$

27) $3\sqrt{16x^4y^4z}$ $\begin{cases} x \in \mathbb{R} \\ y \in \mathbb{R} \\ z \geq 0, \in \mathbb{R} \end{cases}$

$= 3(4) \cdot x^2 \cdot y^2\sqrt{z}$

$= 12x^2y^2\sqrt{z}$

28) $-2\sqrt{48a^3b^4c^2}$ $\begin{cases} a \geq 0, \in \mathbb{R} \\ b \in \mathbb{R} \\ c \in \mathbb{R} \end{cases}$

$= -2\sqrt{16 \cdot 3 \cdot a^2 \cdot a\,(b^2)^2 \cdot c^2}$

$= -2(4) \cdot a \cdot b^2 \cdot c\sqrt{3a}$

$= \boxed{-8ab^2c\sqrt{3a}}$

29) $6\sqrt{75mp^2q^3}$ $\begin{cases} m \geq 0, \in \mathbb{R} \\ p \in \mathbb{R} \\ q \geq 0, \in \mathbb{R} \end{cases}$

$6\sqrt{25 \cdot 3\, m p^2 q^2 \cdot q}$

$= 30pq\sqrt{3mq}$

30) $4\sqrt{36x^2y^3z^4}$ $\begin{cases} x \in \mathbb{R} \\ y \geq 0, \in \mathbb{R} \\ z \in \mathbb{R} \end{cases}$

$= 4(6)(x)(y)(z^2)\sqrt{y}$

$= 24xyz^2\sqrt{y}$

31. $\sqrt{-32y^6}$ <— undefined

If: $= -\sqrt{16 \cdot 2(y^3)^2}$

$= -4y^3\sqrt{2}$, $y \in \mathbb{R}$

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9. Expand and simplify

a) $(2\sqrt{x}-3)(4\sqrt{x}+2)$

$= (2\sqrt{x})(4\sqrt{x}) + 4\sqrt{x} - 12\sqrt{x} - 6$

$= 8x - 8\sqrt{x} - 6$ $\boxed{= 2(4x - 4\sqrt{x} - 3)}$

b) $(\sqrt{x}-2\sqrt{y})(\sqrt{x}+2\sqrt{y}) + (3\sqrt{x}+\sqrt{y})^2$

$= (x - 4y) + 9x + 6\sqrt{x}\sqrt{y} + y$

$\boxed{= 10x - 3y + 6\sqrt{xy}}$

d) $(3\sqrt{x}+\sqrt{y})(3\sqrt{x}-\sqrt{y}) - (\sqrt{x}+5\sqrt{y})^2$

$= (9x - y) - (x + 2(5)\sqrt{xy} + 25y)$

$\boxed{= 8x - 26y - 10\sqrt{xy}}$

10. Rationalize the denominator

a) $\frac{3\sqrt{5}-1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3(5) - \sqrt{5}}{5}$

$\boxed{= \frac{15 - \sqrt{5}}{5}}$

b) $\frac{2\sqrt{3}-7\sqrt{5}}{\sqrt{12}} \cdot \frac{\sqrt{12}}{\sqrt{12}} = \frac{2\sqrt{3}(2\sqrt{3} - 7\sqrt{5})}{12}$

$= \frac{12 - 14\sqrt{15}}{12}$ $\boxed{= 6}$

c) $\frac{5\sqrt{3}+\sqrt{2}}{2\sqrt{6}+4\sqrt{3}} \cdot \frac{2\sqrt{6}-4\sqrt{3}}{2\sqrt{6}-4\sqrt{3}}$

$= \frac{10\sqrt{18} - 60 + 2\sqrt{12} - 4\sqrt{6}}{4(6) - 48}$

$= \frac{30\sqrt{2} - 60 + 4\sqrt{3} - 4\sqrt{6}}{-24}$

$= \frac{15\sqrt{2} - 30 + 2\sqrt{3} - 2\sqrt{6}}{-12}$

d) $\frac{4+2\sqrt{6}}{3\sqrt{2}-4} \cdot \frac{3\sqrt{2}+4}{3\sqrt{2}+4}$

$= \frac{12\sqrt{2} + 16 + 6\sqrt{12} + 8\sqrt{6}}{18 - 16}$

$= \frac{12\sqrt{2} + 16 + 12\sqrt{3} + 8\sqrt{6}}{2}$

$\boxed{= 6\sqrt{2} + 8 + 6\sqrt{3} + 4\sqrt{6}}$

e) $\frac{2\sqrt{5}-\sqrt{3}}{3\sqrt{2}+\sqrt{5}} - \frac{2\sqrt{3}-4\sqrt{6}}{3\sqrt{2}+2\sqrt{3}}$

11. Solve each equation. Remember to check for extraneous roots.

23) $4+\sqrt{-3m+10}=m$ $\quad -3m+10\ge 0;\ m\le 10/3.$

$(\sqrt{-3m+10})^2=(m-4)^2$

$-3m+10=m^2-8m+16$

$0=m^2-5m+6$

$0=(m-3)(m-2)$

$m=2$ or 3.

24) $(x-5)^2=(\sqrt{x+1})^2$ $\quad x+1\ge 0;\ x\ge -1$

$x^2-10x+25=x+1$

$x^2-11x+24=0$.

$x=\frac{11\pm\sqrt{(-11)^2-4(24)}}{2}=\frac{11\pm 5}{2}$ $\quad x_1=\frac{16}{2}=8 \quad x_2=\frac{6}{2}=3$

Check: $(8-5)=\sqrt{8+1}$, $3=3$ ✓ $\quad (3-5)=\sqrt{3+1}$, $-2=2$ ✗

∴ x=3 is an extraneous root.
x = 8 is a solution

25) $n-7=\sqrt{3n-21}$ $\quad 3n-21\ge 0;\ n\ge 7$

$n^2-14n+49=3n-21$

$n^2-17n+70=0$

$n=\frac{17\pm\sqrt{(-17)^2-4(70)}}{2}=\frac{17\pm 3}{2}$

$x_1=10$

$x_2=7$

Check: $(10-7)=\sqrt{30-21}$, $3=3$ ✓

$(7-7)=\sqrt{3(7)-21}$, $0=\sqrt{0}$, $0=0$ ✓

∴ 2 solutions $x=10, 7$.

26) $b-6=\sqrt{18-3b}$

$b^2-12b+36=18-3b$

$b^2-9b+18=0$

$(b-6)(b-3)=0$

$b=6$ or 3.

$18-3b\ge 0$; $b\le 6$

Check: $6-6=\sqrt{18-3(6)}$, $0=0$ ✓

$3-6=\sqrt{18-3(7)}$, $-3=\sqrt{9}$, $-3=3$ ✗

∴ b=3 is extraneous; b=6 is a solution

27) $-3+\sqrt{m+59}=m$ $\quad m+59\ge 0,\ m\ge -59$

$(\sqrt{m+59})^2=(m+3)^2$

$m+59=m^2+6m+9$

$0=m^2+5m-50$

$0=(m+10)(m-5)$

$m=-10$ or 5.

Check: $-3+\sqrt{-10+59}=-10$; $-3+7=-10$; $4=-10$ ✗

$-3+\sqrt{5+59}=5$; $-3+8=5$; $5=5$ ✓

28) $\sqrt{7a-54}-a=-6$

$(\sqrt{7a-54})^2=(-6+a)^2$

$7a-54=a^2-12a+36$

$0=a^2-19a+90$

$0=(a-10)(a-9)$

$a=10$ or 9

$7a-54\ge 0$ $\quad a\ge 54/7$

Check: $7(10)-54-10=-6$; $70-64=-6$; $6=-6$ ✗

$7(9)-54-9=-6$; $63-45=-6$; $18=-6$ ✗

∴ Both extraneous

12. Simplify each expression. All variables represent positive numbers.

a) $\sqrt{x^2-6x+9}-\sqrt{x^2-2x+1}$

$\sqrt{(x-3)(x-3)}-\sqrt{(x-1)(x-1)}$

$=\sqrt{(x-3)^2}-\sqrt{(x-1)^2}$

$=|x-3|-|x-1|$

b) $\sqrt{x^2+4x+4}-\sqrt{x^2+10x+25}$

$=\sqrt{(x+2)^2}-\sqrt{(x+5)^2}$

All positive

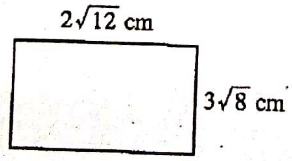
$=|x+2|-|x+5|$

$=x+2-x+5$

$\boxed{=7}$

13. Find the perimeter.

a)



$2(2\sqrt{12})+2(3\sqrt{8})$

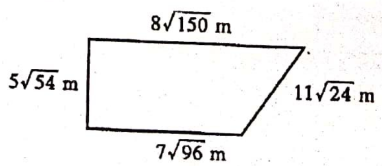
$=4\sqrt{12}+6\sqrt{8}$

$=4(2)\sqrt{3}+6(2)\sqrt{2}$

$=8\sqrt{3}+12\sqrt{2}$

∴ The perimeter is $8\sqrt{3}+12\sqrt{2}$ cm.

b)



$P=5\sqrt{54}+8\sqrt{150}+11\sqrt{24}+7\sqrt{96}$

$=5\sqrt{9}\sqrt{6}+8\sqrt{25}\sqrt{6}+11\sqrt{4}\sqrt{6}+7\sqrt{16}\sqrt{6}$

$=15\sqrt{6}+40\sqrt{6}+22\sqrt{6}+28\sqrt{6}$

$=105\sqrt{6}$

∴ The perimeter is $105\sqrt{6}$ m